Příklad: $f(x,y) = \operatorname{arctg} \frac{x}{y}$ TOT. DIF.?

Kandidát: • $\frac{\partial f}{\partial x}(x,y) = \frac{1}{y} \cdot \frac{1}{1+\left(\frac{x}{y}\right)^2} =$

$= \frac{1}{y} \cdot \frac{y^2}{y^2+x^2} = \frac{y}{y^2+x^2}$

• $\frac{\partial f}{\partial y}(x,y) = \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot x \cdot \frac{-1}{y^2} = \frac{-x}{y^2} \cdot \frac{y^2}{y^2+x^2} =$

$= \frac{-x}{y^2+x^2}$

Pokud existuje $df(x,y)$, pak

$$df(x,y) = \left(\frac{\partial f}{\partial x}(x,y), \frac{\partial f}{\partial y}(x,y)\right) =$$

$$= \left(\frac{y}{y^2+x^2}, \frac{-x}{y^2+x^2}\right)$$

Platí, že pokud $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ jsou v bodě (x,y) spojité, pak $df(x,y)$ existuje.

Neboli ta forma $\left(\frac{y}{x^2+y^2}, \frac{-x}{x^2+y^2}\right)$ je skutečně TD.

3b) Přibližně spočítat $0{,}98^{3{,}04}$.

Víme kolik je 1^3. Obě čísla se mění – a to jinak. $\Rightarrow$ 2 proměnné.

$f(x,y) = x^y$ $\quad f(1,3) = 1$ víme

$= e^{y \cdot \ln x}$ $\quad f(0{,}98\ ,\ 3{,}04) \doteq ?$

$df(1,3) = ?$ $\quad df(1,3)(h_1, h_2)$

$h_1 = -0{,}02, \quad h_2 = 0{,}04$

$\frac{\partial f}{\partial x}(x,y) = y \cdot x^{y-1}$ $\quad \frac{\partial f}{\partial x}(1,3) = 3x^2\Big|_{x=1} = 3$

$\frac{\partial f}{\partial y}(x,y) = e^{y \cdot \ln x} \cdot \ln x = x^y \cdot \ln x$

$\frac{\partial f}{\partial y}(1,3) = 1^3 \cdot \ln 1 = 0$

Tedy $df(1,3) = (3, 0)$

Tedy $df(1,3)(-0{,}02,\ 0{,}04) = (3, 0)\begin{pmatrix} -0{,}02 \\ 0{,}04 \end{pmatrix} = \frac{-6}{100}$

Tedy $\frac{-6}{100}$ je odhad přírůstku f při kroku $h = (h_1, h_2) = (-0{,}02\ ,\ 0{,}04)$.

Tedy $f((1,3) + (h_1, h_2)) = f(0{,}98\ ,\ 3{,}04) \doteq$

$\doteq f(1,3) + df(1,3)(h) = 1 - \frac{6}{100} = 0{,}94$

Tedy $f(0{,}98\ ,\ 3{,}04) \doteq 0{,}94$.

kalkul. $\doteq 0{,}940432\ldots$

4b) $4\sqrt{x^2+y^2} = f(x,y)$ $\qquad A = [3,4,?]$

$A \in G_f \Rightarrow A = [3,4,\ f(3,4)] = [3,4,20]$

Tečná rovina: Obecně má rovnici

$$z = \frac{\partial f}{\partial x}(a,b)\cdot(x-a) + \frac{\partial f}{\partial y}(a,b)\cdot(y-b)+f(a,b).$$

V našem případě: $(a,b)=(3,4)$, $f(a,b)=20$.

$$\frac{\partial f}{\partial x}(x,y) = 4\frac{1}{2\sqrt{x^2+y^2}}\cdot 2x = \frac{4x}{\sqrt{x^2+y^2}}$$

$$\frac{\partial f}{\partial y}(x,y) = \qquad = \frac{4y}{\sqrt{x^2+y^2}}$$

$$\frac{\partial f}{\partial x}(3,4) = \frac{4\cdot 3}{5} = \frac{12}{5} \qquad \frac{\partial f}{\partial y}(3,4) = \frac{16}{5}$$

Tečná rovina ke G_f v A má rci:

$$z = \frac{12}{5}(x-3) + \frac{16}{5}(y-4) + 20$$

$$\frac{12}{5}x + \frac{16}{5}y - z = -20 + \frac{36}{5} + \frac{64}{5} = 0$$

$$\left[\frac{12}{5}x + \frac{16}{5}y - z = 0\right]$$

$$df(1,3) = \begin{pmatrix} 3 & 0 \end{pmatrix}$$

$$= 3dx + 0\,dy$$

Formálně lze interpretovat

$$dx = \begin{pmatrix} 1 & 0 \end{pmatrix} \qquad dy = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

$$f(x,y,z) = (x^{y})^{z} = \left(e^{y \cdot \ln x}\right)^{z} = e^{z \cdot \ln\left(e^{y \ln x}\right)} = e^{zy \ln x} = x^{zy}$$

$$\frac{\partial f}{\partial y}(x,y,z) = e^{zy \ln x} \cdot z \ln x = (x^{y})^{z} \cdot \ln x^{z} \qquad = x^{yz} \ln x^{z} \checkmark$$

$$\frac{\partial f}{\partial z}(x,y,z) = \ldots = (x^{z})^{y} \cdot \ln x^{y} \qquad = x^{yz} \ln x^{y} \checkmark$$

$$\frac{\partial f}{\partial x}(x,y,z) = zy \cdot x^{zy-1} \checkmark$$